Stresses in Large Horizontal Cylindrical Pressure Vessels on Two Saddle Supports

Approximate stresses that exist in cylindrical vessels supported on two saddles at various conditions and design of stiffening for vessels which require it

by L. P. Zick

INTRODUCTION

THE design of horizontal cylindrical vessels with dished heads to resist internal pressure is covered by existing codes. However, the method of support is left pretty much up to the designer. In general the cylindrical shell is made a uniform thickness which is determined by the maximum circumferential stress due to the internal pressure. Since the longitudinal stress is only one-half of this circumferential stress, these vessels have available a beam strength which makes the two-saddle support system ideal for a wide range of proportions. However, certain limitations are necessary to make designs consistent with the intent of the code.

The purpose of this paper is to indicate the approximate stresses that exist in cylindrical vessels supported on two saddles at various locations. Knowing these stresses, it is possible to determine which vessels may be designed for internal pressure alone, and to design structurally adequate and economical stiffening for the vessels which require it. Formulas are developed to cover various conditions, and a chart is given which covers support designs for pressure vessels made of mild steel for storage of liquid weighing 42 lb. per cu. ft.

HISTORY

In a paper[1] published in 1933 Herman Schorer pointed out that a length of cylindrical shell supported by tangential end shears varying proportionately to the sine of the central angle measured from the top of the vessel can support its own metal weight and the full contained liquid weight without circumferential bending moments in the shell. To complete this analysis, rings around the entire circumference are required at the supporting points to transfer these shears to the foundation without distorting the cylindrical shell. Discussions of Schorer's paper by H. C. Boardman and others gave approximate solutions for the half full condition. When a ring of uniform cross section is supported on two vertical posts, the full condition governs the design of the ring if the central angle between the post intersections with the ring is less than 126°, and the half-full condition governs if this angle is more than 126°. However, the full condition governs the design of rings supported directly in or adjacent to saddles.

Mr. Boardman's discussion also pointed out that the heads may substitute for the rings provided the supports are near the heads. His unpublished paper has been used successfully since 1941 for vessels supported on saddles near the heads. His method of analysis covering supports near the heads is included in this paper in a slightly modified form.

Discussions of Mr. Schorer's paper also gave successful and semi-successful examples of unstiffened cylindrical shells supported on saddles, but an analysis is lacking. The semi-successful examples indicated that the shells had actually slumped down over the horns of the saddles while being filled with liquid, but had rounded up again when internal pressure was applied.

Testing done by others[2, 3] gave very useful results in the ranges of their respective tests, but the investigators concluded that analysis was highly indeterminate. In recent years the author has participated in strain gage surveys of several large vessels.[4] A typical test setup is shown in Fig. 1.

In this paper an attempt has been made to produce an approximate analysis involving certain empirical assumptions which make the theoretical analysis closely approximate the test results.

SELECTION OF SUPPORTS

When a cylindrical vessel acts as its own carrying beam across two symmetrically placed saddle supports, one-half of the total load will be carried by each support. This would be true even if one support should

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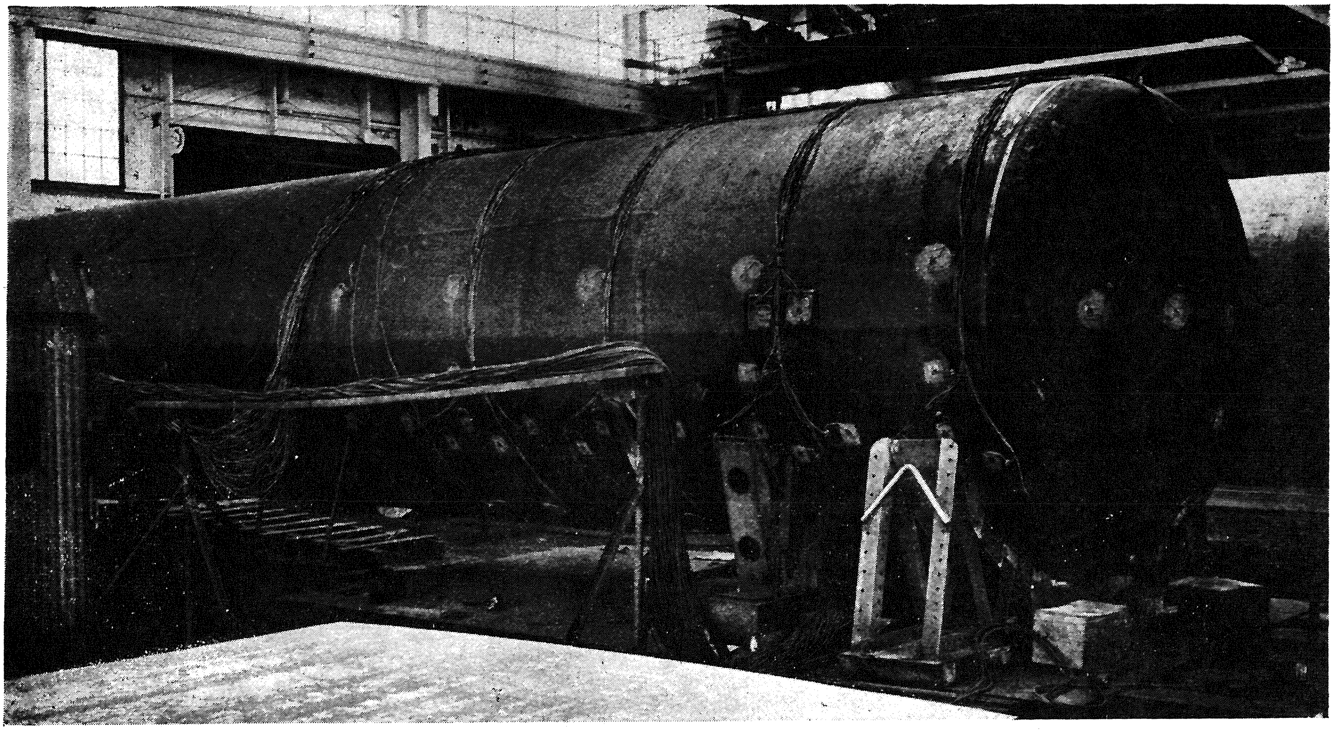

Fig. 1 Strain gage test set up on 30,000 gal. propane tank

settle more than the other. This would also be true if a differential in temperature or if the axial restraint of the supports should cause the vessel acting as a beam to bow up or down at the center. This fact alone gives the two-support system preference over a multiple-support system.

The most economical location and type of support generally depend upon the strength of the vessel to be supported and the cost of the supports, or of the supports and additional stiffening if required. In a few cases the advantage of placing fittings and piping in the bottom of the vessel beyond the saddle will govern the location of the saddle.

The pressure-vessel codes limit the contact angle of each saddle to a minimum of 120° except for very small vessels. In certain cases a larger contact angle should be used. Generally the saddle width is not a controlling factor; so a nominal width of 12 in. for steel or 15 in. for concrete may be used. This width should be increased for extremely heavy vessels, and in certain cases it may be desirable to reduce this width for small vessels.

Thin-wall vessels of large diameter are best supported near the heads provided they can support their own weight and contents between supports and provided the heads are stiff enough to transfer the load to the saddles. Thick-wall vessels too long to act as simple beams are best supported where the maximum longitudinal bending stress in the shell at the saddles is nearly equal to the maximum longitudinal bending stress at mid-span, provided the shell is stiff enough to resist this bending and to transfer the load to the saddles. Where the stiffness required is not available in the shell alone, ring stiffeners must be added at or near the saddles. Vessels must also be rigid enough to support normal external loads such as wind.

Figure 2 indicates the most economical locations and types of supports for large steel horizontal pressure vessels on two supports. A liquid weight of 42 lb. per cu. ft. was used because it is representative of the volatile liquids usually associated with pressure vessels.

Where liquids of different weights are to be stored or where different materials are to be used, a rough design may be obtained from the chart and this design should be checked by the applicable formulas outlined in the following sections. Table I outlines the coefficients to be used with the applicable formulas for various support types and locations. The notation used is listed at the end of the paper under the heading Nomenclature.

MAXIMUM LONGITUDINAL STRESS

The cylindrical shell acts as a beam over the two supports to resist by bending the uniform load of the vessel and its contents. The equivalent length of the vessel (see Figs. 2 and 3) equals $(L + (4H/3))$, closely, and the total weight of the vessel and its contents equals $2Q$. However, it can be shown that the liquid weight in a hemispherical head adds only a shear load at its junction with the cylinder. This can be approximated for heads where $H \leqq R$ by representing the pressure on the head and the longitudinal stress as a clockwise couple on the head shown at the left of Fig. 3. Therefore the vessel may be taken as a beam loaded as shown in Fig. 3; the moment diagram determined by statics is also shown. Maximum moments occur at the mid-span and over the supports.

Tests have shown that except near the saddles a cylindrical shell just full of liquid has practically no circumferential bending moments and therefore behaves as a beam with a section modulus $I/c = \pi r^2 t$.

However, in the region above each saddle circumferential bending moments are introduced allowing the unstiffened upper portion of the shell to deflect, thus making it ineffective as a beam. This reduces the effective cross section acting as a beam just as though the shell were split along a horizontal line at a level above the

Table I—Values of Coefficients in Formulas for Various Support Conditions

Saddle angle, θ	Maximum long. bending stress, Min. K_1*	Tangent. shear, K_2	Circumf. stress top of saddle, K_3†	Additional head stress, K_4	Ring compres. in shell, K_5	Ring stiffeners—Circumf. bending, K_6	Ring stiffeners—Direct stress, K_7	Tension across saddle, K_8
			Shell unstiffened					
120°	0.63 (A/L = 0.09)	1.171	0.0528	...	0.760	...	...	0.204
150°	0.55 (A/L = 0.11)	0.799	0.0316	...	0.673	...	...	0.260
			Shell stiffened by head, $A \leq R/2$					
120°	1.0 (A/L = 0)	0.880	0.0132	0.401	0.760	...	...	0.204
150°	1.0 (A/L = 0)	0.485	0.0079	0.297	0.673	...	...	0.260
			Shell stiffened by ring in plane of Saddle					
120°	0.23 (A/L = 0.193)	0.319	...	...	...	0.0528	0.340	0.204
150°	0.23 (A/L = 0.193)	0.319	...	...	...	0.0316	0.303	0.260
			Shell stiffened by rings adjacent to saddle					
120°	0.23 (A/L = 0.193)	1.171	...	...	0.760	0.0577	0.263	0.204
150°	0.23 (A/L = 0.193)	0.799	...	...	0.673	0.0353	0.228	0:260

* See Fig. 5, which plots K_1 against A/L, for values of K_1 corresponding to values of A/L not listed in table.
† See Fig. 7.

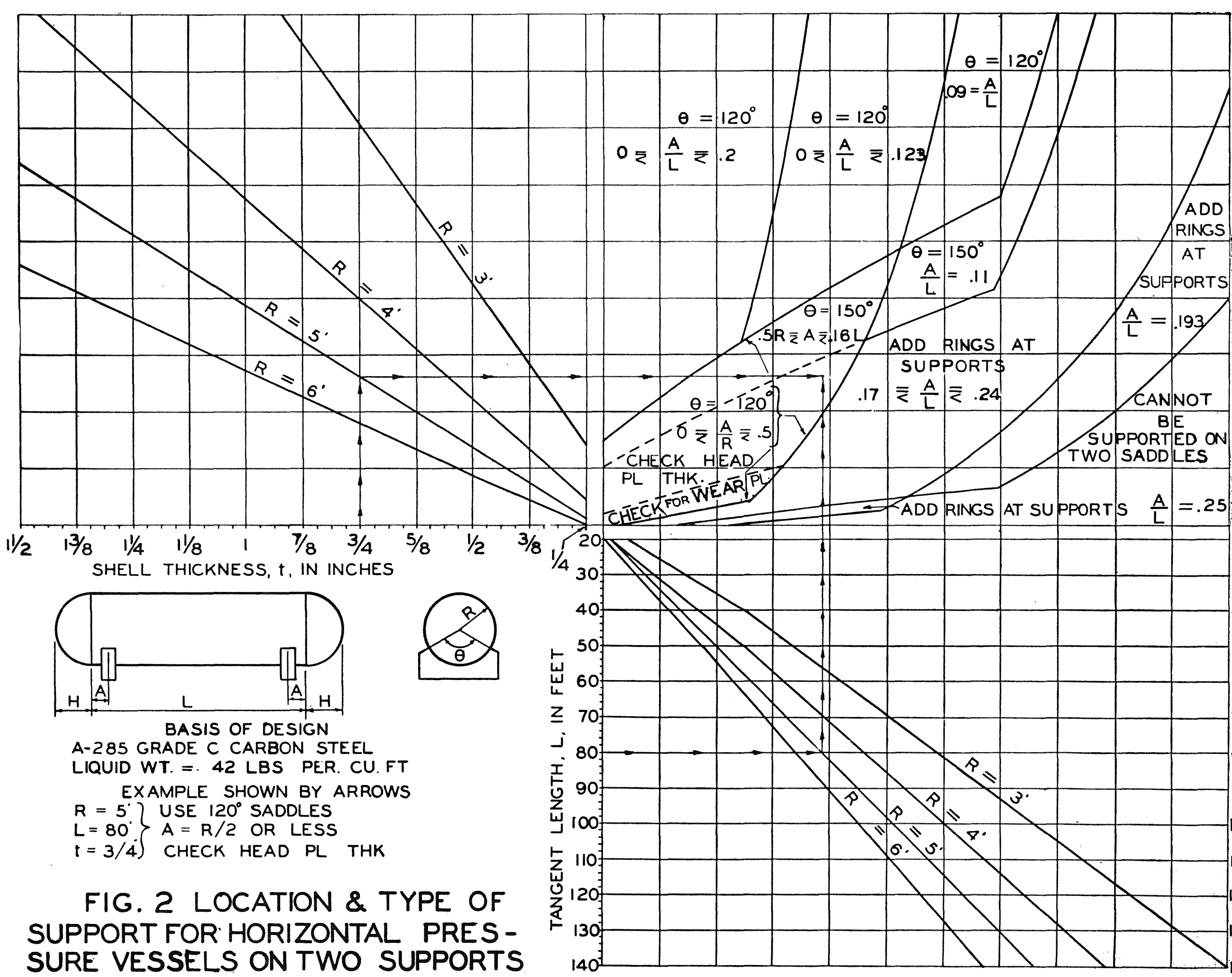

FIG. 2 LOCATION & TYPE OF SUPPORT FOR HORIZONTAL PRESSURE VESSELS ON TWO SUPPORTS

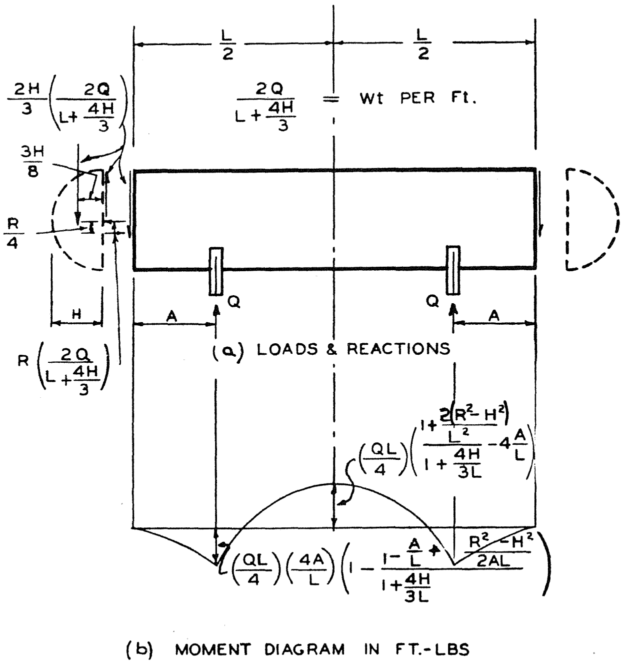

Fig. 3 Cylindrical shell acting as beam over supports

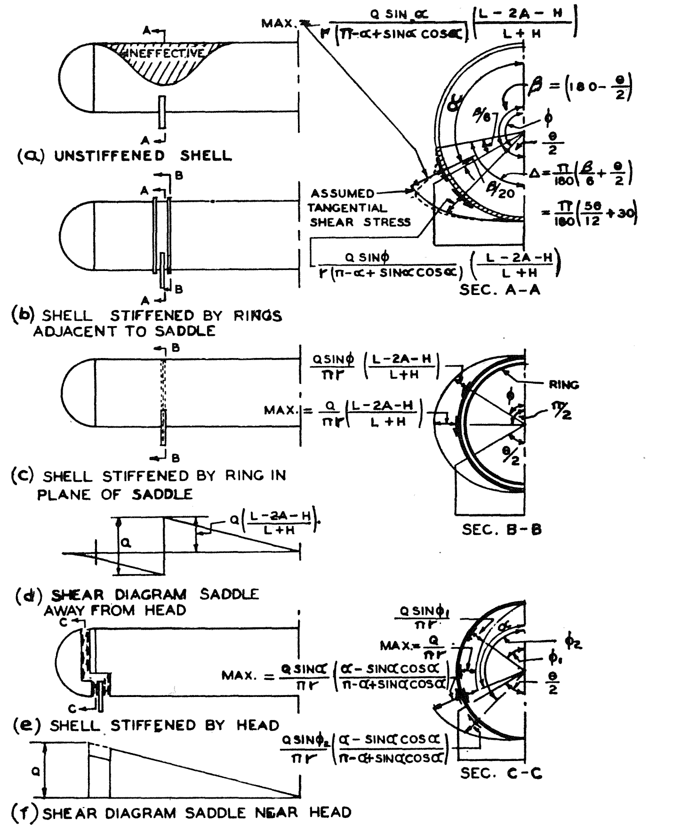

Fig. 4 Load transfer to saddle by tangential shear stresses in cylindrical shell

saddle. (See Fig. 4 (*a*).) If this effective arc is represented by 2Δ (Δ in radians) it can be shown that the section modulus becomes:

$$I/c = \pi r^2 t \left(\frac{\Delta + \sin \Delta \cos \Delta - 2 \dfrac{\sin^2 \Delta}{\Delta}}{\pi \left(\dfrac{\sin \Delta}{\Delta} - \cos \Delta \right)} \right)$$

Strain gage studies indicate that this effective arc is approximately equal to the contact angle plus one-sixth of the unstiffened shell as indicated in Section *A-A* of Fig. 4. Of course, if the shell is stiffened by a head or complete ring stiffener near the saddle the effective arc, 2Δ, equals the entire cross section, and $I/c = \pi r^2 t$.

Since most vessels are of uniform shell thickness, the design formula involves only the maximum value of the longitudinal bending stress. Dividing the maximum moment by the section modulus gives the maximum axial stress in lb. per sq. in. in the shell due to bending as a beam, or

$$S_1 = \pm \frac{3K_1QL}{\pi r^2 t}$$

K_1 is a constant for a given set of conditions, but actually varies with the ratios A/L and $H/L \leqq R/L$ for different saddle angles. For convenience, K_1 is plotted in Fig. 5 against A/L for various types of saddle supports, assuming conservative values of $H = 0$ when the mid-span governs and $H = R$ when the shell section at the saddle governs. A maximum value of $R/L = 0.09$ was assumed because other factors govern the design for larger values of this ratio. As in a beam the mid-span governs for the smaller values of A/L and the shell section at the saddle governs for the larger values of A/L; however, the point where the bending stress in the shell is equal at mid-span and at the saddle varies with the saddle angle because of the reduced effective cross section.

This maximum bending stress, S_1, may be either tension or compression. The tension stress when combined with the axial stress due to internal pressure should not exceed the allowable tension stress of the material times the efficiency of the girth joints. The compression stress should not exceed one half of the compression yield point of the material or the value given by

Allowable Compression Stress

$$S_1 \lessgtr \left(\frac{E}{29}\right)(t/r)[2 - (2/3)(100)(t/r)]$$

which is based upon the accepted formula for buckling of short steel cylindrical columns. The compression

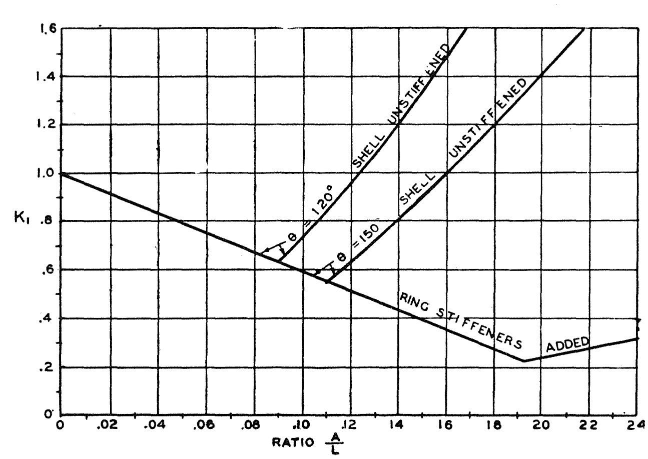

Fig. 5 Plot of longitudinal bending-moment constant, K_1

stress is not a factor in a steel vessel where $t/r \geqq 0.005$ and the vessel is designed for internal pressure.

TANGENTIAL SHEAR STRESS

Figure 4 (*d*) shows the total shear diagram for vessels supported in saddles away from the heads.

Where the shell is held round, the tangential shearing stresses vary directly with the sine of the central angle ϕ, as shown in Section *B-B* of Fig. 4, and the maximum occurs at the equator.

However, if the shell is free to deform above the saddle, the tangential shearing stresses act on a reduced effective cross section and the maximum occurs at the horn of the saddle. This is approximated by assuming the shears continue to vary as the sin ϕ but only act on twice the arc given by $(\theta/2 + \beta/20)$ or $(\pi - \alpha)$ as shown in Section *A-A* of Fig. 4. The summation of the vertical components of these assumed shears must equal the maximum total shear.

The maximum tangential shear stress will occur on the center side of the saddle provided the saddle is beyond the influence of the head but not past the quarter point of the vessel. Then with saddles away from the heads the maximum shear stress in lb. per sq. in. is given by

$$S_2 = \frac{K_2 Q}{rt}\left(\frac{L - 2A - H}{L + H}\right)$$

Values of K_2 listed in Table I for various types of supports are obtained from the expressions given for the maximum shears in Fig. 4.

Figure 4 (*f*) indicates the total shear diagram for vessels supported on saddles near the heads. In this case the head stiffens the shell in the region of the saddle. This causes most of the tangential shearing stress to be carried across the saddle to the head, and then the load is transferred back to the head side of the saddle by tangential shearing stresses applied to an arc slightly larger than the contact angle of the saddle. Section *C-C* of Fig. 4 indicates this shear distribution; that is, the shears vary as the sin ϕ and act downward above angle α and act upward below angle α. The summation of the downward vertical components must balance the summation of the upward vertical components. Then with saddles at the heads the maximum shear stress in lb. per sq. in. is given by

$$S_2 = \frac{K_2 Q}{rt}$$

in the shell, or

$$S_2 = \frac{K_2 Q}{rt_h}$$

in the head.

Values of K_2 given in Table I for different size saddles at the heads are obtained from the expression given for the maximum shear stress in Section *C-C* of Fig. 4.

The tangential shear stress should not exceed 0.8 of the allowable tension stress.

CIRCUMFERENTIAL STRESS AT HORN OF SADDLE

In the plane of the saddle the load must be transferred from the cylindrical shell to the saddle. As was pointed out in the previous section the tangential shears adjust their distribution in order to make this transfer with a minimum amount of circumferential bending and distortion. The evaluation of these shears was quite empirical except for the case of the ring stiffener in the plane of the saddle. Evaluation of the circumferential bending stresses is even more difficult.

Starting with a ring in the plane of the saddle, the shear distribution is known. The bending moment at any point above the saddle may be computed by any of the methods of indeterminate structures. If the ring is assumed uniform in cross section and fixed at the horns of the saddles, the moment, M_ϕ, in in.-lb. at any point A is given by:

$$M_\phi = \frac{Qr}{\pi}\left\{\cos\phi + \frac{\phi}{2}\sin\phi - \frac{3}{2}\frac{\sin\beta}{\beta} + \frac{\cos\beta}{2} - \frac{1}{4}\left(\cos\phi - \frac{\sin\beta}{\beta}\right) \times \left[9 - \frac{4 - 6\left(\frac{\sin\beta}{\beta}\right)^2 + 2\cos^2\beta}{\frac{\sin\beta}{\beta}\cos\beta + 1 - 2\left(\frac{\sin\beta}{\beta}\right)^2}\right]\right\}$$

This is shown schematically in Fig. 6. Note that β must be in radians in the formula.

The maximum moment occurs when $\phi = \beta$. Substituting β for ϕ and K_6 for the expression in the brackets divided by π, the maximum circumferential bending moment in in.-lb. is

$$M_\beta = K_6 Qr$$

When the shell is supported on a saddle and there is no ring stiffener the shears tend to bunch up near the horn of the saddle, so that the actual maximum circumferential bending moment in the shell is considerably less than M_β as calculated above for a ring stiffener in the plane of the saddle. The exact analysis is not known; however, stresses calculated on the assumption

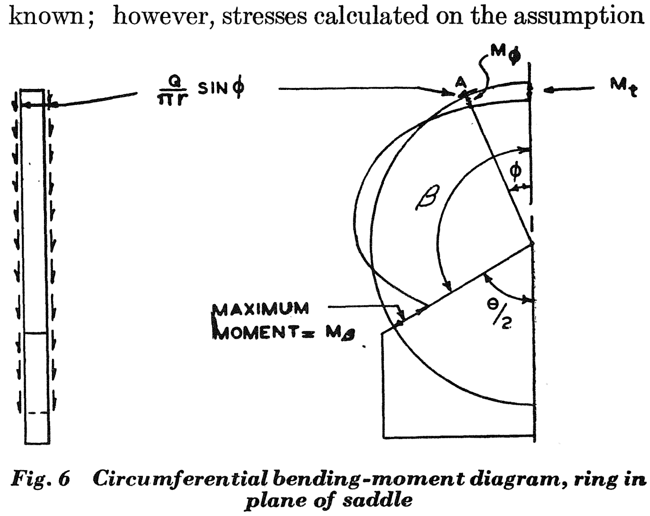

Fig. 6 Circumferential bending-moment diagram, ring in plane of saddle

that a wide width of shell is effective in resisting the hypothetical moment, M_β, agree conservatively with the results of strain gage surveys. It was found that this effective width of shell should be equal to 4 times the shell radius or equal to one-half the length of the vessel, whichever is smaller. It should be kept in mind that use of this seemingly excessive width of shell is an artifice whereby the hypothetical moment M_β is made to render calculated stresses in reasonable accord with actual stresses.

When the saddles are near the heads, the shears carry to the head and are then transferred back to the saddle. Again the shears tend to concentrate near the horn of the saddle. Because of the relatively short stiff members this transfer reduces the circumferential bending moment still more.

To introduce the effect of the head the maximum moment is taken as

$$M_\beta = K_3 Q r$$

Where K_3 equals K_6 when A/R is greater than 1. Values of K_3 are plotted in Fig. 7 using the assumption that this moment is divided by four when A/R is less than 0.5.

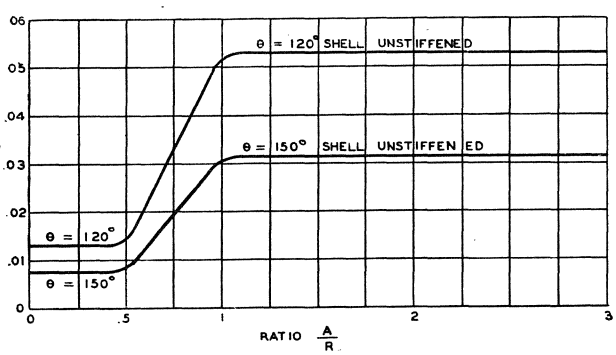

Fig. 7 Plot of circumferential bending-moment constant, K_3

The change in shear distribution also reduces the direct load at the horns of the saddle; this is assumed to be $Q/4$ for shells without added stiffeners. However, since this load exists, the effective width of the shell which resists this direct load is limited to that portion which is stiffened by the contact of the saddle. It is assumed that $5t$ each side of the saddle acts with the portion directly over the saddle.

Internal pressure stresses do not add directly to the local bending stresses, because the shell rounds up under pressure. Therefore the maximum circumferential combined stress in the shell is compressive, occurs at the horn of the saddle, and is due to local bending and direct stress. This maximum combined stress in lb. per sq. in. is given by

$$S_3 = -\frac{Q}{4t(b + 10t)} - \frac{3K_3Q}{2t^2}, \quad \text{if } L \geqq 8R$$

or

$$S_3 = -\frac{Q}{4t(b + 10t)} - \frac{12K_3QR}{Lt^2}, \quad \text{if } L < 8R$$

It seems reasonable to allow this combined stress to be equal to 1.25 times the tension allowable provided the compressive strength of the material equals the tensile strength. In the first place when the region at the horn of the saddle yields, it acts as a hinge, and the upper portion of the shell continues to resist the loads as a two-hinged arch. There would be little distortion until a second point near the equator started to yield. Secondly, if rings are added to reduce this local stress, a local longitudinal bending stress occurs at the edge of the ring under pressure.[5] This local stress would be 1.8 times the design ring stress if the rings were infinitely rigid. Weld seams in the shell should not be located near the horn of the saddle where the maximum moment occurs.

EXTERNAL LOADS

Long vessels with very small t/r values are susceptible to distortion from unsymmetrical external loads such as wind. It is assumed that vacuum relief valves will be provided where required; so it is not necessary to design against a full vacuum. However, experience indicates that vessels designed to withstand 1 lb. per sq. in. external pressure can successfully resist external loads encountered in normal service.

Assume the external pressure is 1 lb. per sq. in. in the formulas used to determine the sloping portion of the external pressure chart in the 1950 A.S.M.E. Unfired Pressure Vessel Code. Then when the vessel is unstiffened between the heads, the maximum length in feet between stiffeners (the heads) is given approximately by

$$L + {}^2/_3H = \frac{E\sqrt{rt}}{52.2}\left(\frac{t}{r}\right)^2$$

When ring stiffeners are added to the vessel at the supports, the maximum length in feet between stiffeners is given by

$$L - 2A = \frac{E\sqrt{rt}}{52.2}\left(\frac{t}{r}\right)^2$$

ADDITIONAL STRESS IN HEAD USED AS STIFFENER

When the head stiffness is utilized by placing the saddle close to the heads, the tangential shear stresses cause an additional stress in the head which is additive to the pressure stress. Referring to Section *C-C* of Fig. 4, it can be seen that the tangential shearing stresses have horizontal components which would cause varying horizontal tension stresses across the entire height of the head if the head were a flat disk. The real action in a dished head would be a combination of ring action and direct stress; however, for simplicity the action on a flat disk is considered reasonable for design purposes.

Assume that the summation of the horizontal components of the tangential shears is resisted by the vertical cross section of the flat head at the center line, and assume that the maximum stress is 1.5 times the average stress. Then the maximum additional stress in the head in lb. per sq. in. is given by

$$S_4 = \frac{3Q}{8rt_h}\left(\frac{\sin^2 \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right)$$

or

$$S_4 = \frac{K_4 Q}{rt_h}$$

This stress should be combined with the stress in the head due to internal pressure. However, it is recommended that this combined stress be allowed to be 25% greater than the allowable tension stress because of the nature of the stress and because of the method of analysis.

WEAR PLATES—RING COMPRESSION IN SHELL OVER SADDLE

Figure 8 indicates the saddle reactions, assuming the surfaces of the shell and saddle are in frictionless contact without attachment. The sum of the assumed tangential shears on both edges of the saddle at any point A is also shown in Fig. 8. These forces acting on the shell band directly over the saddle cause ring compression in the shell band. Since the saddle reactions are radial, they pass through the center O. Taking moments about point O indicates that the ring compression at any point A is given by the summation of the tangential shears between α and ϕ.

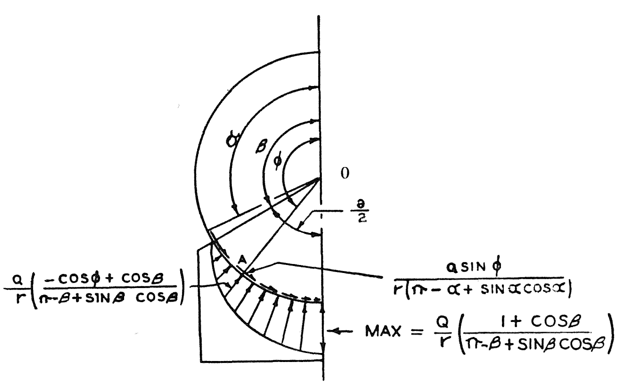

Fig. 8 Loads and reactions on saddles

This ring compression is maximum at the bottom, where $\phi = \pi$. Again a width of shell equal to $5t$ each side of the saddle plus the width of the saddle is assumed to resist this force. Then the stress in lb. per sq. in. due to ring compression is given by

$$S_5 = \frac{Q}{t(b + 10t)}\left(\frac{1 + \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right)$$

or

$$S_5 = \frac{K_5 Q}{t(b + 10t)}$$

The ring compression stress should not exceed one-half of the compression yield point of the material.

The stress may be reduced by attaching a wear plate some what larger than the surface of the saddle to the shell directly over the saddle. The thickness t may be taken as the combined thickness of the shell and the wear plate in the formulas for S_5 and S_3 provided the width of the added plate equals $(b + t)$ and provided the plate extends $r/10$ in. above the horn of the saddle.

DESIGN OF RING STIFFENERS

When the saddles must be located away from the heads and when the shell alone cannot resist the circumferential bending, ring stiffeners should be added at or near the supports. Because the size of rings involved does not warrant further refinement, the formulas developed in this paper assume that the added rings are continuous with a uniform cross section. The ring stiffener must be attached to the shell, and the portion of the shell reinforced by the stiffener plus a width of shell equal to $5t$ each side may be assumed to act with each stiffener.

When n stiffeners are added directly over the saddle as shown in Fig. 4 (c), the tangential shear distribution is known. The equation for the resulting bending moment at any point was developed previously, and the resulting moment diagram is shown in Fig. 6. The maximum moment occurs at the horn of the saddle and is given in in.-lb. for each stiffener by

$$M_\beta = K_6 \frac{Qr}{n}$$

Knowing the maximum moment M_β and the moment at the top of the vessel, M_t, the direct load at the point of maximum moment may be found by statics. Then the direct load at the top of the saddle is given in pounds by

$$P_\beta = \frac{Q}{\pi n}\left[\frac{\beta \sin \beta}{2(1 - \cos \beta)} - \cos \beta\right] + \frac{\cos \beta}{r(1 - \cos \beta)}(M_\beta - M_t)$$

or

$$P_\beta = K_7 \frac{Q}{n}$$

If n stiffeners are added adjacent to the saddle as shown in Fig. 4 (b), the rings will act together and each will be loaded with shears distributed as in Section B-B on one side but will be supported on the saddle side by a shear distribution similar to that shown in Section A-A. Conservatively, the support may be assumed to be tangential and concentrated at the top of the saddle. This is shown schematically in Fig. 9; the resulting bending moment diagram is also indicated. This bending moment in in.-lb. at any point A above the top of the saddle is given by

$$M_\phi = \frac{Qr}{2\pi n}\left\{\frac{\pi - \beta}{\sin \beta} - \phi \sin \phi - \cos \phi\,[3/2 + (\pi - \beta)\cot \beta]\right\}$$

For the range of saddle angles considered M_ϕ is maximum near the equator where $\phi = \rho$. This moment and the direct stress may be found using a procedure similar to that used for the stiffener in the plane of the saddle. Substituting ρ for ϕ and K_6 for the expression in the brackets divided by 2π, the maximum moment in each ring adjacent to the saddle is given in in.-lb. by

$$M_\rho = K_6 \frac{Qr}{n}$$

Knowing the moments M_ρ and M_t, the direct load at ρ may be found by statics and is given by

$$P_\rho = \frac{Q}{\pi n}\left[\frac{\rho \sin \rho}{2(1 - \cos \rho)} - \cos \rho\right] - \frac{\cos \rho}{r(1 - \cos \rho)}(M_\rho + M_t)$$

or

$$P_\rho = K_7 \frac{Q}{n}$$

Then the maximum combined stress due to liquid load in each ring used to stiffen the shell at or near the saddle is given in lb. per sq. in. by

$$S_6 = -\frac{K_7 Q}{na} \pm \frac{K_6 Qr}{nI/c}$$

where a = the area and I/c = the section modulus of the cross section of the composite ring stiffener. When a ring is attached to the inside surface of the shell directly over the saddle or to the outside surface of the shell adjacent to the saddle, the maximum combined stress is compression at the shell. However, if the ring is attached to the opposite surface, the maximum combined stress may be either compression in the outer flange due to liquid or tension at the shell due to liquid and internal pressure.

The maximum combined compression stress due to liquid should not exceed one-half of the compression yield point of the material. The maximum combined tension stress due to liquid and pressure should not exceed the allowable tension stress of the material.

DESIGN OF SADDLES

Each saddle should be rigid enough to prevent the separation of the horns of the saddle; therefore the saddle should be designed for a full water load. The horn of the saddle should be taken at the intersection of the outer edge of the web with the top flange of a steel saddle. The minimum section at the low point of either a steel or concrete saddle must resist a total force, F, in pounds, equal to the summation of the horizontal components of the reactions on one-half of the saddle. Then

$$F = Q\left[\frac{1 + \cos \beta - 1/2 \sin^2 \beta}{\pi - \beta + \sin \beta \cos \beta}\right] = K_8 Q$$

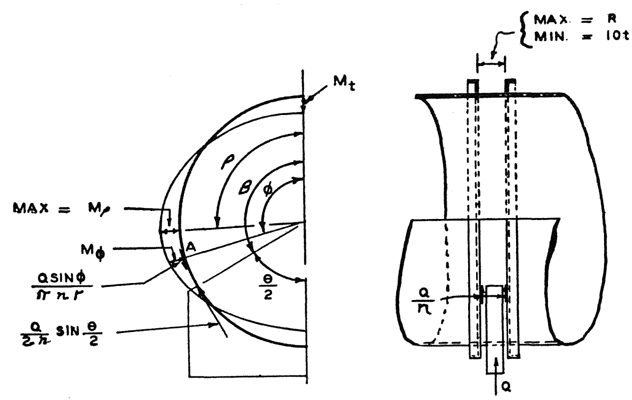

Fig. 9 Circumferential bending-moment diagram, stiffeners adjacent to saddle

The effective section resisting this load should be limited to the metal cross section within a distance equal to $r/3$ below the shell. This cross section should be limited to the reinforcing steel within the distance $r/3$ in concrete saddles. The average stress shoule not exceed two-thirds of the tension allowable of the material. A low allowable stress is recommended because the effect of the circumferential bending in the shell at the horn of the saddle has been neglected.

The upper and lower flanges of a steel saddle should be designed to resist bending over the web(s), and the web(s) should be stiffened according to the A.I.S.C. Specifications against buckling. The contact area between the shell and concrete saddle or between the metal saddle and the concrete foundation should be adequate to support the bearing loads.

Where extreme movements are anticipated or where the saddles are welded to the shell, bearings or rockers should be provided at one saddle. Under normal conditions a sheet of elastic waterproof material at least 1/4 in. thick between the shell and a concrete saddle will suffice.

Nomenclature

Q = load on one saddle, lb. Total load = $2Q$.
L = tangent length of the vessel, ft.
A = distance from center line of saddle to tangent line, ft.
H = depth of head, ft.
R = radius of cylindrical shell, ft.
r = radius of cylindrical shell, in.
t = thickness of cylindrical shell, in.
t_h = thickness of head, in.
b = width of saddle, in.
F = force across bottom of saddle, lb.
S_1, S_2, etc. = calculated stresses, lb. per sq. in.
K_1, K_2, etc. = dimensionless constants for various support conditions.
M_ϕ, M_β, etc. = circumferential bending moment due to tangential shears, in.-lb.
θ = angle of contact of saddle with shell, degrees.
β = $\left(180 - \frac{\theta}{2}\right)$ = central angle from vertical to horn of saddle, in degrees (except as noted).
Δ = $\frac{\pi}{180}\left(\frac{\theta}{2} + \frac{\beta}{6}\right) = \frac{\pi}{180}\left(\frac{5\theta}{12} + 30\right)$. 2Δ = arc, in radians, of unstiffened shell in plane of saddle effective against bending.

$\alpha \;= \pi - \frac{\pi}{180}\left(\frac{\theta}{2}+\frac{\beta}{20}\right)$ = the central angle, in radians, from the vertical to the assumed point of maximum shear in unstiffened shell at saddle.

ϕ = any central angle measured from the vertical, in radians.

ρ = central angle from the upper vertical to the point of maximum moment in ring located adjacent to saddle, in radians.

E = modulus of elasticity of material, lb. per sq. in.

I/c = section modulus, in.^{3}

n = number of stiffeners at each saddle.

a = cross-sectional area of each composite stiffener, sq. in.

P_ρ, P_β = the direct load in lb. at the point of maximum moment in a stiffening ring.

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SHELL 중앙부
M1 = 3*K1*Q*L(ft)

Appendix

The formulas developed by outline in the text are developed mathematically here under headings corresponding to those of the text. The pertinent assumptions and statements appearing in the text have not been repeated.

MAXIMUM LONGITUDINAL STRESS

Referring to Fig. 3, the bending moment in ft.-lb. at the saddle is

$$\frac{2Q}{L+\frac{4H}{3}}\left[\frac{2HA}{3}+\frac{A^2}{2}-\frac{R^2-H^2}{4}\right] = QA\left[1-\frac{1-\frac{A}{L}+\frac{R^2-H^2}{2AL}}{1+\frac{4H}{3L}}\right]$$

SHELL 중앙부
M2 =

Referring to Section A-A of Fig. 4 the c the effective arc $= r\dfrac{\sin\,\Delta}{\Delta}$. If δ equals any central angle measured from the bottom, the moment of inertia is

$$2r^3t\int_0^\Delta\left(\cos^2\delta - 2\cos\delta\frac{\sin\,\Delta}{\Delta}+\frac{\sin^2\Delta}{\Delta^2}\right)d\delta =$$

$$2r^3t\left[1/2\sin\delta\cos\delta+\frac{\delta}{2}-\frac{2\sin\delta\sin\Delta}{\Delta}+\frac{\sin^2\Delta}{\Delta^2}\delta\right]_0^\Delta =$$

$$r^3t\left[\sin\Delta\cos\Delta+\Delta-2\frac{\sin^2\Delta}{\Delta}\right]$$

The section modulus for the tension side of the equivalent beam is

$$r^2t\left[\frac{\Delta+\sin\Delta\cos\Delta-2\frac{\sin^2\Delta}{\Delta}}{\frac{\sin\,\Delta}{\Delta}-\cos\Delta}\right]$$

Then the stress in the shell at the saddle in lb. per sq. in. is given by

$$S_1 = \frac{3QL}{\pi r^2 t}\left[\frac{4A}{L}\left(1-\frac{1-\frac{A}{L}+\frac{R^2-H^2}{2AL}}{1+\frac{4H}{3L}}\right)\times\frac{\pi\left(\frac{\sin\,\Delta}{\Delta}-\cos\,\Delta\right)}{\Delta+\sin\,\Delta\cos\,\Delta-2\frac{\sin^2\Delta}{\Delta}}\right]$$

or

$$S_1 = \frac{3K_1QL}{\pi r^2 t}$$

where

$$K_1 = \left[\frac{\pi\left(\frac{\text{Sin}\,\Delta}{\Delta}-\cos\,\Delta\right)}{\Delta+\sin\,\Delta\cos\,\Delta-2\frac{\sin^2\Delta}{\Delta}}\right]\times\left[\frac{4A}{L}\left(1-\frac{1-\frac{A}{L}+\frac{R^2-H^2}{2AL}}{1+\frac{4H}{3L}}\right)\right]$$

The bending moment in ft.-lb. at the mid-span is

$$\frac{2Q}{L+\frac{4H}{3}}\left[\frac{(L-2A)^2}{8}-\frac{2HA}{3}-\frac{A^2}{2}+\frac{R^2-H^2}{4}\right] = \frac{QL}{4}\left(\frac{1+2\frac{R^2-H^2}{L^2}}{1+\frac{4H}{3L}}-4\frac{A}{L}\right)$$

The section modulus $= \pi r^2 t$, and

$$S_1 = \frac{3K_1QL}{\pi r^2 t}$$

where

$$K_1 = \left(\frac{1+2\frac{R^2-H^2}{L^2}}{1+\frac{4H}{3L}}-4\frac{A}{L}\right)$$

Tangential Shear Stress

Section B-B of Fig. 4 indicates the plot of the shears adjacent to a stiffener. The summation of the vertical components of the shears on each side of the stiffener must equal the load on the saddle Q. Referring to Fig. 4 (d) the sum of the shears on both sides of the stiffener at any point is $Q\sin\phi/\pi r$. Then the summation of the vertical components is given by

$$2\int_0^\pi\frac{Q\sin^2\phi}{\pi r}r d\phi = \frac{2Q}{\pi}\left[\frac{\phi}{2}-\frac{\sin\phi\cos\phi}{2}\right]_0^\pi = Q$$

The maximum shear stress occurs at the equator when $\sin\phi = 1$ and $K_2 = 1/\pi = 0.319$.

Section A-A of Fig. 4 indicates the plot of the shears in an unstiffened shell. Again this summation of the vertical components of the shears on each side of the saddle must equal the load on the saddle. Then the total shear at any point is

$$\frac{Q \sin \phi}{r(\pi - \alpha + \sin \alpha \cos \alpha)}$$

and the summation of the vertical components is given by

$$2\int_{\alpha}^{\pi} \frac{Q \sin^2 \phi}{r(\pi - \alpha + \sin \alpha \cos \alpha)} r d\phi = Q\left[\frac{\phi - \sin \phi \cos \phi}{\pi - \alpha + \sin \alpha \cos \alpha}\right]_{\alpha}^{\pi} = Q$$

The maximum shear occurs where $\phi = \alpha$ and

$$K_2 = \frac{\sin \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}$$

Section C-C of Fig. 4 indicates the shear transfer across the saddle to the head and back to the head side of the saddle. Here the summation of the vertical components of the shears on arc α acting downward must equal the summation of the vertical component of the shears on the lower arc $(\pi - \alpha)$ acting upward. Then

$$2\int_{0}^{\alpha} \frac{Q \sin^2 \phi_1}{\pi r} r d\phi_1 = 2\int_{\alpha}^{\pi} \frac{Q \sin^2 \phi_2}{\pi r}\left[\frac{\alpha - \sin \alpha \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right] r d\phi$$

or

$$\frac{2Q}{\pi}\left[\frac{\phi_1}{2} - \frac{\sin \phi_1 \cos \phi_1}{2}\right]_0^{\alpha} = \frac{2Q}{\pi}\left[\frac{\alpha - \sin \alpha \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right]\left[\frac{\phi_2}{2} - \frac{\sin \phi_2 \cos \phi_2}{2}\right]_{\alpha}^{\pi}$$

Finally

$$\frac{Q}{\pi}(\alpha - \sin \alpha \cos \alpha) = \frac{Q}{\pi}(\alpha - \sin \alpha \cos \alpha)$$

The maximum shear occurs where $\phi_2 = \alpha$ and

$$K_2 = \frac{\sin \alpha}{\pi}\left[\frac{\alpha - \sin \alpha \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right]$$

CIRCUMFERENTIAL STRESS AT HORN OF SADDLE

See under the heading Design of Ring Stiffeners.

Additional Stress in Head Used as Stiffener

Referring to Section C-C of Fig. 4, the tangential shears have horizontal components which cause tension across the head. The summation of these components on the vertical axis is

$$\int_0^{\alpha} \frac{Q}{\pi r} \sin \phi_1 \cos \phi_1 r d\phi_1 - \int_{\alpha}^{\pi} \frac{Q}{\pi r} \sin \phi_2 \cos \phi_2 \left[\frac{\alpha - \sin \alpha \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right] r d\phi_2 =$$

$$\frac{Q}{\pi}\left\{\left[\frac{\sin^2 \phi_1}{2}\right]_0^{\alpha} - \left[\frac{\alpha - \sin \alpha \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right]\left[\frac{\sin^2 \phi_2}{2}\right]_{\alpha}^{\pi}\right\} = \frac{Q}{2}\left(\frac{\sin^2 \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right)$$

Then assuming this load is resisted by $2rt_h$ and that the maximum stress is 1.5 times the average

$$S_4 = \frac{K_4 Q}{rt_h}$$

where

$$K_4 = \frac{3}{8}\left(\frac{\sin^2 \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right)$$

WEAR PLATES

The ring compression at any point in the shell over the saddle is given by the summation of the tangential shears over the arc $= (\phi - \alpha)$ shown in Section A-A or C-C of Fig. 4 or in Fig. 8. Then

$$-\int_{\alpha}^{\phi} \frac{Q \sin \phi_2}{\pi r}\left(\frac{\alpha - \sin \alpha \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right) r d\phi_2 - \int_{\alpha}^{\phi} \frac{Q \sin \phi_1}{\pi r} r d\phi_1 = -\int_{\alpha}^{\phi} \frac{Q \sin \phi_2 d\phi_2}{(\pi - \alpha + \sin \alpha \cos \alpha)} =$$

$$-\left[\frac{Q}{\pi - \alpha + \sin \alpha \cos \alpha}\right]\left[\cos \phi_2\right]_{\alpha}^{\phi} = Q\left[\frac{-\cos \phi + \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right]$$

The ring compression becomes a maximum in the shell at the bottom of the saddle. Or if $\phi = \pi$ this expression becomes

$$Q\left[\frac{1 + \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right]$$

Then

$$K_5 = \left[\frac{1 + \cos \alpha}{\pi - \alpha + \sin \alpha \cos \alpha}\right]$$

DESIGN OF RING STIFFENERS

Stiffener in Plane of Saddle

Referring to Fig. 6, the arch above the horns of the saddle resists the tangential shear load. Assuming this arch fixed at the top of the saddles, the bending moment may be found using column analogy.

If the arch is cut at the top, the static moment at any point A is

$$M_S = \frac{Qr}{\pi}\int_0^{\phi}(\sin \phi_1 - \sin \phi_1 \cos \phi_1 \cos \phi - \sin^2 \phi_1 \sin \phi)\, d\phi_1$$

$$= \frac{Qr}{\pi}\left[-\cos \phi_1 - \frac{\cos \phi}{2}\sin^2 \phi_1 + \frac{1}{2}\sin \phi \sin \phi_1 \cos \phi_1 - \frac{\phi_1 \sin \phi}{2}\right]_0^{\phi} = \frac{Qr}{\pi}\left[1 - \cos \phi - \frac{\phi}{2}\sin \phi\right]$$

Then the M_S/EI diagram is the load on the analogous column.

The area of this analogous column is

$$a_1 = 2\int_0^\beta \frac{r}{EI}\,d\phi = \frac{2\beta r}{EI}$$

The centroid is $\sin\beta/\beta\, r$, and the moment of inertia about the horizontal axis is

$$I_h = 2\int_0^\beta \left(\cos\phi - \frac{\sin\beta}{\beta}\right)^2 \frac{r^3}{EI}\,d\phi =$$

$$\frac{2r^3}{EI}\left[\frac{1}{2}\sin\phi\cos\phi + \frac{1}{2}\phi - \frac{2\sin\phi\sin\beta}{\beta} + \frac{\phi\sin^2\beta}{\beta^2}\right]^\beta =$$

$$\frac{r^3}{EI}\left[\sin\beta\cos\beta + \beta - \frac{2\sin^2\beta}{\beta}\right]$$

The load on the analogous column is

$$q = 2\int_0^\beta \frac{M_S}{EI} r\,d\phi = \frac{2Qr^2}{\pi EI}\int_0^\beta \left(1 - \cos\phi - \frac{\phi}{2}\sin\phi\right)d\phi$$

$$q = \frac{2Qr^2}{\pi EI}\left[\phi - \sin\phi - \frac{\sin\phi}{2} + \frac{\phi\cos\phi}{2}\right]_0^\beta =$$

$$\frac{Qr^2}{\pi EI}\left[2\beta - 3\sin\beta + \beta\cos\beta\right]$$

The moment about the horizontal axis is

$$M_h = -2\int_0^\beta \frac{M_S}{EI}\left(\cos\phi - \frac{\sin\beta}{\beta}\right)r^2 d\phi =$$

$$-\frac{Qr^3}{\pi EI}\int_0^\beta \Big(2\cos\phi - 2\cos^2\phi - \phi\sin\phi\cos\phi - \frac{\sin\beta}{\beta}(2 - 2\cos\phi - \phi\sin\phi)\Big)d\phi =$$

$$-\frac{Qr^3}{\pi EI}\Big[2\sin\phi - \cos\phi\sin\phi - \phi - \frac{\sin\phi\cos\phi}{4} + \frac{\phi}{4} - \frac{\phi\sin^2\phi}{2} - \frac{\sin\beta}{\beta}(2\phi - 2\sin\phi - \sin\phi + \phi\cos\phi)\Big]_0^\beta =$$

$$\frac{Qr^3}{\pi EI}\left[\frac{9}{4}\sin\beta\cos\beta + \frac{3}{4}\beta - \frac{3\sin^2\beta}{\beta} + \frac{\beta\sin^2\beta}{2}\right]$$

Then the indeterminate moment is

$$Mi = \frac{q}{a_1} - \frac{M_h Y}{I_h} = \frac{Qr}{\pi}\left\{\frac{2\beta - 3\sin\beta + \beta\cos\beta}{2\beta} - \frac{Y}{4r}\left[\frac{9\beta\sin\beta\cos\beta + 3\beta^2 - 12\sin^2\beta + 2\beta^2\sin^2\beta}{\beta\sin\beta\cos\beta + \beta^2 - 2\sin^2\beta}\right]\right\}$$

The distance from the neutral axis to point A is given by

$$Y = \left(\cos\phi - \frac{\sin\beta}{\beta}\right)r$$

Finally, the combined moment is given by

$$M_\phi = -M_S + M_i = \frac{Qr}{\pi}\left\{\cos\phi + \frac{\phi}{2}\sin\phi - \frac{3}{2}\frac{\sin\beta}{\beta} + \frac{\cos\beta}{2} - 1/4\left(\cos\phi - \frac{\sin\beta}{\beta}\right)\times\left[9 - \frac{4 - 6\left(\frac{\sin\beta}{\beta}\right)^2 + 2\cos^2\beta}{\frac{\sin\beta}{\beta}\cos\beta + 1 - 2\left(\frac{\sin\beta}{\beta}\right)^2}\right]\right\}$$

This is the maximum when $\phi = \beta$; then

$$M_\beta = \frac{Qr}{\pi}\left\{\frac{\beta\sin\beta}{2} - \frac{3}{4}\cos\beta + \frac{3}{4}\frac{\sin\beta}{\beta} + \left(\frac{\cos\beta - \frac{\sin\beta}{\beta}}{4}\right)\left[\frac{4 - 6\left(\frac{\sin\beta}{\beta}\right)^2 + 2\cos^2\beta}{\frac{\sin\beta\cos\beta}{\beta} + 1 - 2\left(\frac{\sin\beta}{\beta}\right)^2}\right]\right\}$$

Finally $\qquad M_\beta = K_6 Qr$

Because of symmetry the shear stress is zero at the top of the vessel; therefore, the direct load in the ring at the top of the vessel, P_t, may be found by taking moments on the arc β about the horn of the saddle. Then

$$(1 - \cos\beta)rP_t = \frac{Qr}{\pi}\left[1 - \cos\beta - \frac{\beta}{2}\sin\beta\right] - (M_\beta - M_t)$$

$$P_t = \frac{Q}{\pi}\left[1 - \frac{\beta\sin\beta}{2(1 - \cos\beta)}\right] - \frac{1}{r(1 - \cos\beta)}(M_\beta - M_t)$$

The direct load, P_β, at $\phi = \beta$, the point of maximum moment may be found by taking moments about the center. Then

$$r(P_\beta + P_t) = \frac{Qr}{\pi}(1 - \cos\beta) - (M_\beta - M_t)$$

Substituting the value above for P_t and solving for P_β gives

$$P_\beta = \frac{Q}{\pi}\left[\frac{\beta\sin\beta}{2(1 - \cos\beta)} - \cos\beta\right] + \frac{\cos\beta}{r(1 - \cos\beta)}(M_\beta - M_t)$$

or $\qquad P_\beta = K_7 Q$

where

$$K_7 = \frac{1}{\pi}\left[\frac{\beta\sin\beta}{2(1 - \cos\beta)} - \cos\beta\right] + \frac{\cos\beta}{Qr(1 - \cos\beta)}(M_\beta - M_t)$$

If the rings are adjacent to the saddle, K_6 and K_7 may be found in a similar manner, except that the static structure would become the entire ring split at the top and loaded as indicated in Fig. 9.

Design of Saddles

The summation of the horizontal components of the radial reactions on one-half of the Saddle shown in Fig. 8 must be resisted by the saddle at $\phi = \pi$. Then this horizontal force is given by

$$F = \int_\beta^\pi \frac{Q(-\cos\phi\sin\phi + \cos\beta\sin\phi)}{r(\pi - \beta + \sin\beta\cos\beta)} r\,d\phi =$$

$$Q\left[\frac{-1/2\sin^2\phi - \cos\phi\cos\beta}{\pi - \beta + \sin\beta\cos\beta}\right]_\beta^\pi =$$

$$Q\left[\frac{1 + \cos\beta - 1/2\sin^2\beta}{\pi - \beta + \sin\beta\cos\beta}\right]$$

Then $\qquad K_8 = \dfrac{1 + \cos\beta - 1/2\sin^2\beta}{\pi - \beta + \sin\beta\cos\beta}$

The bending at the horn would change the saddle reaction distribution, and increase this horizontal force.